Design Optimization Methods for Genomic DNA Tiling Arrays

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Abstract

A recent development in microarray construction entails the unbiased coverage, or tiling, of non-repetitive genomic DNA for the experimental identification of unannotated transcribed sequences and regulatory elements. A central issue in designing tiling arrays is one of sequence similarity, as significant experimental cross-hybridization can result from the incorporation of non-unique probe sequences. Many genomes contain thousands of repetitive elements that must also be identified and omitted from the array design. Due to the fragmentation introduced by repeats, the problem of obtaining adequate sequence coverage increases with the sizes of subsequence tiles that are to be included on the array. Here we describe the general problem of designing arrays with tiles of varying sizes, and discuss the issues that arise when tiling with shorter and longer sequences. The general problem of sequence tiling can be framed as finding an optimal partitioning of non-repetitive subsequence fragments, or tiles, for a given range of sizes on a DNA sequence containing repetitive and non-repetitive regions. Exact solutions to the tiling problem can become computationally infeasible when applied to large genomes, but we develop successive optimizations that allow their practical implementation. In particular, we present an efficient method for rapidly determining the degree of similarity of many oligonucleotide sequences in large genomes, and two algorithms for finding an optimal tile path over genomic sequences using longer sequence tiles. The first algorithm, a dynamic programming approach, finds an optimal tiling in linear time and linear space; the second applies a heuristic search to reduce the space complexity to a constant requirement. We apply these methods to several complete eukaryotic genomes, illustrating the degree to which optimal tiling differs from a trivial partitioning of the sequence. The improvement in non-repetitive sequence coverage is most pronounced in complex mammalian genomes, which exhibit a much higher degree of sequence fragmentation due to increased repeat content.

DNA microarrays have become ubiquitous in genomic research as tools for the large-scale analysis of gene expression. The design of DNA microarrays has generally focused on the measurement of mRNA transcript levels from annotated genes, represented either by PCR products comprising entire cDNA sequences (Schena et al. 1995), or by short oligonucleotides complementary to internal regions of spliced messages (Lipshutz et al. 1999). Microarrays of this design allow the simultaneous interrogation of thousands of nucleotide sequences, providing a genome-wide snapshot of transcriptional activity. Since its introduction, microarray technology has advanced to a degree that currently accommodates enough individual array features to represent all the annotated genes in a mammalian genome.

A recent trend in genomics has involved the development of "tiling" arrays: microarrays that represent a complete non-repetitive tile path over a chromosome or locus, irrespective of any genes that may be annotated in that region (Figure 1, left). This unbiased representation of genomic DNA has enabled the discovery of many novel transcribed sequences (Kapranov et al. 2002; Rinn et al. 2002; Yamada et al. 2003; Kampa et al. 2004, Bertone et al. 2004; Cheng et al. 2005), as well as the global identification of transcription factor binding sites (Ren et al. 2000; Iyer et al. 2001; Lee et al. 2002; Horak et al. 2002b; Martone et al. 2003; Cawley et al. 2004; Euskirchen et al. 2004, Odom et al. 2004; Kim et al. 2005).

In addition to coding and regulatory sequences, genomes contain repetitive elements that have been introduced and replicated in high copy number over evolutionary time. The frequency and diversity of these repeat sequences increases with the size and complexity of higher eukaryotic chromosomes, accounting for approximately 45% of the total nucleotide content of mammalian genomes. In selecting sequences to be represented on a microarray, the exclusion of repetitive elements and other non-unique sequences is a primary goal. The reasons for this are twofold: first, microarray features whose sequences contain repeats present highly redundant hybridization targets; these contribute non-specific background signals that mask the fluorescence resulting from specific probe hybridization. Second, the inclusion of repeats can significantly increase the number of DNA sequences assigned to array features. This additional sequence content is superfluous, resulting in the suboptimal utilization of the available features on a given array platform.

When representing genomic DNA with short oligonucleotides, near-optimal coverage of the non-repetitive sequence can be achieved in a relatively straightforward manner (Figure 2B), although a number of important factors should be considered for probe selection. A more problematic situation arises when tile sizes increase, such as when selecting suitable targets for PCR amplification. For this application it is necessary to derive a tile path of larger sequence fragments from the non-repetitive component of the genome (Figure 2C). Small PCR products can be difficult to resolve in a high-throughput setting, while fragments of several kilobases (kb) in length can limit the precise identification of hybridizing sequences. Balancing these criteria to select appropriate target sequences while avoiding repetitive elements presents a challenging optimization problem.

Here we discuss a number of issues central to the problem of partitioning non-repetitive sequences using a range of tile sizes, and present several optimization approaches suitable for both oligonucleotide- and amplicon-based microarray applications. A web resource has also been developed for genomic sequence tiling, accessible at tiling.gersteinlab.org, that coordinates a suite of programs to generate optimal tile paths at various resolutions from user-provided DNA sequence files.

METHODS

Tiling discontiguous genome sequences

Genomic tiling arrays are intended to maximally cover a span of non-repetitive DNA with representative sequence fragments, or tiles, whose sizes fall within a prescribed range. The number of repetitive elements included in the tile path should be minimized, while the sequence is partitioned into the fewest number of tiles that can maximally cover the non-repetitive DNA. The sequences included on the array are either PCR-amplified and deposited mechanically onto glass slides via contact printing (Schena et al. 1995), or partitioned further and represented as oligonucleotides that may be printed mechanically or synthesized *in situ* using photolithographic (Lipshutz et al. 1999; Nuwaysir et al. 2002) or piezoelectric (Hughes et al. 2001) technologies (Table 1). Since the number of available features on a given microarray platform is often dependent on production costs, in practice an optimal tiling solution should arrive at the fewest number of non-repetitive tiles whose lengths approach a pre-determined upper bound.

Sequence similarity and single-copy tiling

Genome sequences are not random and therefore contain many redundant subsequences. For the design of tiling arrays it is essential to identify and eliminate non-unique sequences in order to reduce the potential for cross-hybridization of sequences originating from elsewhere in the genome. For shorter sequence tiles, robust methods to address the problem of sequence similarity can be developed, thereby generating a single-copy tile path to be represented on the array (Figure 2A). To implement this approach we compute the degree of similarity of any given oligonucleotide sequence in a large genome.

This problem can be stated as follows: Given a genomic sequence and an oligomer of length n, find all oligomers in the sequence differing from the input in no more than m places. In theory, we need only create a direct hash table of each sequence to a list of all subsequence occurrences. However, the space required to implement the hash quickly becomes impractical. With 4^n possible oligomer sequences, a hash of size 14 requires 1 gigabyte of storage, in addition to the space needed to store each of the possible index coordinates of the input sequence (another gigabyte for large chromosomes). These requirements impose a practical limitation on the size of hash tables such that $n \leq 14$. This is insufficient for most microarray applications where oligonucleotide sizes are typically ≥ 25 nt.

To work around these memory constraints and account for sequence mismatches, we adopt a BLAST-like scheme similar to that described in Wang and Seed (2003). A hash table is created based on oligomers of size $k < n$. When considering a given oligonucleotide sequence, we look up each of the oligonucleotide's $n - k$ substrings of length k, extending each hit to full length as dictated by the substring's position in the oligonucleotide and comparing it to the input sequence. In doing so we can also allow for mismatches, knowing that we will detect all oligonucleotides with no more than two mismatches to the input so long as $k \geq (n - m) / (m + 1)$. Given a random model of a chromosome of length c, a substring of length k will have an expected $c / 4^k$ matches, each of which can be processed in constant time. In such a model our algorithm runs in an expected time of $O((n - k) / 4^k)$.

Repeat identification and low-complexity filtering

On a global scale, the implementation of exact methods to address the problem of sequence similarity becomes impractical. Therefore, approximations are found using tools designed to identify known repetitive elements in genomic DNA through sequence comparison. For the purpose of designing microarrays, it is necessary to locate repetitive elements in genomic sequence with local alignment methods (Smith and Waterman 1981; Altschul et al. 1990). This is most easily accomplished through the use of software such as RepeatMasker (Smit and Green, unpublished), CENSOR (Jurka et al. 1996), Tandem Repeats Finder (Benson 1999) and RECON (Bao and Eddy 2002). Of these, RepeatMasker is widely used and is capable of identifying repeats in a variety of genomes using a database of well-characterized families of repetitive elements (Jurka 2000).

In addition to identifying instances of canonical repeat families, it is often desirable to screen genomic DNA for low-complexity sequences: stretches of polypurine/polypyrimidine bases, or regions of extremely high A/T or G/C content. RepeatMasker is able to filter some low-complexity DNA by default; more extensive filtering is often performed using programs such as DUST (Tatusov and Lipman) and NSEG (Wooton and Federhen, 1993). DUST is included as a component of the NCBI BLAST distribution; NSEG is a member of the SEG family of programs and affords more flexible control over low-complexity filtering by using an information entropy-based model of sequence analysis.

Genomic DNA representation with short sequence tiles

When developing gene-based microarrays with short oligonucleotides, one or more probes are typically selected to represent each gene (Tomiuk and Hofmann 2001; Xu et al. 2002). These are designed to be highly specific to the target gene, to anneal within a suitable affinity range, to occur within annotated exons so that they will hybridize to the mature spliced transcript (Wang and Seed 2003), and are typically positioned proximal to the 3' end of the gene to increase the likelihood of detecting partially reverse-transcribed messages.

Designing oligonucleotide tiling arrays constitutes a different problem than selecting oligonucleotides for gene-based arrays, primarily because end-to-end or overlapping tile layouts present fewer options with regard to sequence selection. A number of factors should be considered when tiling genomic DNA with oligonucleotides, including tiling resolution, uniqueness of oligonucleotide sequences, and hybridization affinity. Subdividing contiguous genomic DNA in a naïve, end-to-end fashion offers little opportunity to select optimal probe sequences because the aim is to cover the non-repetitive regions using predetermined spacing constraints. However, several strategies can be used to improve both the annealing specificity and thermodynamic properties of oligonucleotides selected for tiling arrays.

Tiling resolution

An important factor in microarray design entails determining how the remaining non-repetitive DNA should be subdivided and how densely it should be represented by oligonucleotide probes. The serial placement of oligonucleotides along segments of non-repetitive genomic DNA can either be contiguous, covering all of the available sequence, or discontiguous, where gaps of a predetermined size range are allowed between adjacent probes (Figure 1, upper right). This determination should be made according to the type of experiment for which the microarray is intended, and what kind of biological information the array is capable of measuring given a particular experimental sample. In the case of ChIP-chip experiments, chromatin-immunoprecipited DNA is hybridized to an intergenic microarray to locate transcription factor binding sites (Horak and Snyder 2002; Cawley et al. 2004). The immunoprecipitated DNA is sonicated prior to hybridization to shear the molecules into smaller fragments; even so, fragments smaller than approximately 500 bp will be largely unaffected by sonication. Since the sample DNA comprises a population of molecules whose sizes will generally exceed 500 bp, it is reasonable to represent the genomic sequence with oligonucleotide probes spaced under 500 bp apart. Although closer probe spacing will yield more precise hybridization data, larger gaps are still appropriate for ChIP-chip experiments because this layout will ensure adequate hybridization to the sample DNA.

For the fine-resolution mapping of transcribed sequences, much closer probe spacing is required. Because a large fraction of the coding sequences in many eukaryotes span only tens of nucleotides, most of these would elude detection if the genomic sequence is tiled with large gaps. Further, if the experiment is intended to measure exon-intron boundaries, it may be desirable to cover the genomic DNA with multiple oligonucleotides such that the starting position of each probe is shifted by several nucleotides in order to overlap the previous oligonucleotide's coordinates (Figure 1, lower right). Although this strategy increases the tiling resolution, the number of probes required will eventually occupy many more features on the array. It is therefore important to select the desired tiling resolution in a manner that considers the intended microarray platform and optimizes the use of the available array elements.

Oligonucleotide probes that are selected for microarray applications are typically short (25 – 80nt) and uniform in length. These assumptions allow the non-repetitive regions to be tiled by adopting a naïve approach in which the sequences are subdivided into fixed-size partitions. There will naturally be many cases where the oligomer length does not divide evenly into the size of a non-repetitive sequence fragment and the remainder is therefore omitted from the tile path. However, the resulting loss in sequence coverage is inconsequential given the typically short length of the oligonucleotides. In these situations, it is desirable to adjust the placement of oligonucleotides in order to bias the sequence selection toward the optimal criteria, thereby reducing the potential for cross-hybridization to sequences elsewhere in the genome.

Thermodynamic properties of oligonucleotide probes

A third factor concerns the selection of oligonucleotide sequences for tiling arrays based on their predicted hybridization affinities (SantaLucia 1998). When representing individual genes with oligonucleotides, careful consideration is made to select sequences unique to each gene, having thermodynamic characteristics that are optimal for hybridization. For sequences longer than 13nt, hybridization affinity can be approximated by calculating the melting temperature of the DNA duplex using the standard formula:

$$\text{Tm} = 64.9 + 41(n_\text{G} + n_\text{C} - 16.4)/(n_\text{A} + n_\text{T} + n_\text{G} + n_\text{C})$$

where $n_{[A,C,G,T]}$ indicates the number of instances of each nucleotide present in the DNA sequence. For more precise calculations, we can use a base-stacking approach that takes the exact sequence into account rather than the overall nucleotide composition (Rychlik and Rhoads 1989):

$$\text{Tm} = [\Delta H(\text{kcal/°C*Mol})/\Delta S + R \ln([\text{oligo}]/2)] - 273.15\text{°C}$$

where ΔH is the enthalpy of base stacking interactions, ΔS is the entropy of base stacking, [oligo] indicates the oligonucleotide concentration, and R is the universal gas constant 1.987 Cal/°C*Mol.

Considering these criteria, it is useful to shift the placement of oligonucleotides within each region of non-repetitive DNA in order to reduce the variability of the melting temperatures associated with each probe sequence. In the case of spaced oligo tiling an individual probe is selected from within each available region such that the calculated Tm is closest to the optimal temperature. For overlapping tiling designs either the entire set of oligos can be shifted together such that their aggregate Tm is optimized, or the previous approach can be taken and the available regions for oligo placement simply overlap with adjacent regions instead of considering gaps between them.

Optimizing sequence coverage with longer tiles

Representing genomic DNA with tiles of increasing length involves a number of challenges beyond oligonucleotide selection. Sequences of several hundred base pairs are typically amplified by PCR and therefore must conform to certain properties to facilitate high-throughput amplification. Typically, the size distribution of sequences amenable to both PCR amplification and microarray analysis falls between 300 bp and 1.5kb. Although it is feasible to amplify sequence fragments far exceeding this upper limit, it becomes difficult to determine the locations of hybridizing sequences within larger fragments. Conversely, amplifying thousands of small sequence fragments complicates the production of large-scale projects.

With regard to sequence tiling, a repeat-masked genome sequence can be viewed as containing two categories of nucleotide information: 1) that which comprises the coding, regulatory and intergenic sequences located in euchromatic regions, together viewed as non-repetitive DNA (*nr*DNA), and 2) that which belongs to repetitive elements and low-complexity regions (*rp*DNA). Tiling of repeat-masked sequences can therefore be viewed as a two-class partitioning problem: Given a sequence with some subwords identified as repeat nucleotides and the remaining subwords composed of non-repetitive nucleotides, the sequence is partitioned into non-overlapping tiles of either type such that the total amount of non-repetitive sequence coveredis maximized, while the number of repetitive nucleotides included in the resulting tile path is minimized.

The repetitive elements present in most eukaryotic genomes introduce a high degree of fragmentation of the non-repetitive DNA. Avoiding repeats and targeting only the remaining sequence fragments over 300bp in size results in suboptimal coverage of the non-repetitive DNA (Figure 3). In order to improve the sequence coverage, strategies must be devised to recover some of the non-repetitive fragments that are too small to be efficiently amplified. This can be accomplished by strategically incorporating short repeat elements that lie between these non-repetitive sequences, effectively joining the adjacent fragments into larger contiguous tiles (Figure 4). The methods proposed below are designed to obtain optimal tile paths for repeat-masked genomes, maximizing the coverage of non-repetitive DNA while minimizing the number of repetitive elements included in the resulting sequences.

Algorithms for optimal sequence tiling

Scoring potential tile paths

Given a sequence of nucleotides $S_{1..n}$, we would like to find an optimal tile path (possibly not unique) comprising a set of non-overlapping tiles, potentially separated by excluded regions, that maximizes a scoring function V over all possible tile paths, given by

$$V\left[TilePath\left\{S_{1..n}\right\}\right] = \sum_{i=1}^{n} w_i - mC,$$

where w_i is the weight associated with the ith nucleotide, m is the number of tiles and C is the cost for opening a tile (in this way, fewer longer tiles are favored over the creation of many smaller ones). For a given tile path each nucleotide in the sequence is either in a tile (which have weights w_{nr}^{T} and w_{rp}^{T} for non-repetitive and repetitive nucleotides, respectively) or in an excluded region (which have weights w_{nr}^{X} and w_{rp}^{X} for non-repetitive and repetitive nucleotides, respectively). Thus

$$w_i^{T \text{ or } X} = \begin{cases} w_{nr}^{T \text{ or } X} & \text{if nonrepetitive} \\ w_{rp}^{T \text{ or } X} & \text{if repetitive} \end{cases}.$$

We can also use the scoring function V to evaluate the score of either an individual tile $T_{i..j}$ or an excluded region $X_{i..j}$,

$$V\left[T_{i..j}\right] = \sum_{k=i}^{j} w_k^T - C, \quad V\left[X_{i..j}\right] = \sum_{k=i}^{j} w_k^X .$$

Therefore the scoring function evaluated over an entire tile path is the sum of all scores for individual tiles and excluded regions,

$$V\left[TilePath\left\{S_{1..n}\right\}\right] = \sum_{\{T_a\}} V\left[T_a\right] + \sum_{\{X_a\}} V\left[X_a\right] ,$$

where $\{T_a\}$ is a set of all tiles in the tile path and $\{X_a\}$ is analogously defined. A brute force algorithm would enumerate all tile paths to find an optimal solution; however, this approach would take exponential time to compute. We impose an additional constraint, that tiles are restricted to lengths between a lower bound l and an upper bound u. Given this constraint, the algorithm we present here solves the problem in linear time.

A dynamic programming solution

Dynamic programming has been successfully applied many times in sequence analysis. Examples include alignment methods (Needleman and Wunsch 1970; Smith and Waterman 1981; Gotoh 1982), gene prediction (Gelfand and Roytberg 1993; Snyder and Stormo 1993) and RNA secondary structure prediction (Zuker and Sankoff 1984). The key idea behind dynamic programming is the reuse of intermediate results. This is usually accomplished by breaking down an exponential search space into subparts, which are evaluated and whose results are tabulated for reuse. The analysis of large search spaces can then be done in polynomial time.

The main iteration of the algorithm can be described as follows: at an intermediate step in the computation we have evaluated the optimal tile paths and their associated scores for all subsequences $S_{1..1}$ to $S_{1..(k-1)}$. In order to find an optimal tile path for the subsequence $S_{1..k}$, for each $i \in [\max(1, k-u), \max(1, k-l)]$ we compute the score for the tile path consisting of the optimal tile path from $1..i$ and the tile $T_{(i+1)..k}$ using the score of the optimal tile path from $1..i$ and $V[T_{(i+1)..k}]$. Similarly, we also evaluate the score of the tile path consisting of the optimal solution from $1..(k-1)$ and the excluded region $X_{k..k}$ (the kth nucleotide). The optimal tile path for $S_{1..k}$ is then one of the preceding tile paths having the maximal score. This tile path and its associated score are then stored and the algorithm proceeds to the next nucleotide in the sequence, $k+1$. A schematic of the algorithm appears below.

Given optimal tiles paths for all subsequences $S_{1..1}$ to $S_{1..(k-1)}$ and associated scores $V\left[OptimalTilePath\left\{S_{1..1}\right\}\right]$ to $V\left[OptimalTilePath\left\{S_{1..(k-1)}\right\}\right]$:

STEP 1: For each $i \in \left[\max\left(1,k-u\right),\max\left(1,k-1\right)\right]$ we construct the following tile path:

$$TilePath\left\{S_{1..k}\right\} = OptimalTilePath\left\{S_{1..i}\right\} \cup T_{(i+1)..k}$$

and compute its score

$$V\left[TilePath\left\{S_{1..k}\right\}\right] = V\left[OptimalTilePath\left\{S_{1..i}\right\}\right] + V\left[T_{(i+1)..k}\right]$$

We also construct an additional tile path

$$TilePath\left\{S_{1..k}\right\} = OptimalTilePath\left\{S_{1..(k-1)}\right\} \cup X_{k..k}$$

and compute its score

$$V\left[TilePath\left\{S_{1..k}\right\}\right] = V\left[OptimalTilePath\left\{S_{1..(k-1)}\right\}\right] + V\left[X_{k..k}\right]$$

STEP 2: From the preceding tile paths computed in Step 1, we select one having the maximal score and store it as *OptimalTilePath* $\{S_{1..k}\}$, along with its associated score.
STEP 3: Repeat for subsequence $S_{1..(k+1)}$.

We used the algebraic dynamic programming (ADP) framework (Giegerich et al. 2000) to recursively construct all possible partitionings and apply the scoring scheme to each solution. Since many partitionings share common subpartitionings, we can tabulate their scores for reuse instead of recomputing them (Figure 5). Without the tile length constraints, the time and space complexity of this approach would be $O(n^2)$, which is inherent in the ADP framework implementation. Given these constraints, the algorithm runs in linear time and space, specifically $O((u - l)n)$.

A linear-time, constant-space solution

The dynamic programming algorithm computes an optimal tiling solution over the target sequence. In practice, however, the time and space required to process real genomic DNA sequences preclude the use of this approach for large eukaryotic chromosomes (spanning up to ~250 Mb). Here we present an alternative method which traverses the sequence in a single pass, placing tiles according to local constraints instead of considering every possible tiling solution. In contrast to the dynamic programming algorithm, the result of this approach partitions the sequence into alternating included regions $I_{i..j}$ and excluded regions $X_{i..j}$. A post-processing step is then required to subdivide the included regions into individual tiles $T_{i..j}$ satisfying the length constraints.

The scores for included and excluded regions are given by

$$V\left[I_{i..j}\right] = \sum_{k=i}^{j} w_k^I , \qquad V\left[X_{i..j}\right] = \sum_{k=i}^{j} w_k^X ,$$

where the weights corresponding to included regions are the same as those for the tiles in the dynamic programming algorithm ($w_k^I = w_k^T$). Note that the score for included regions does not account for the tile cost C.

The algorithm partitions the sequence and outputs the region boundaries as processing continues. The sequence is scanned one nucleotide at a time, with the current position denoted by i. During the main iteration we keep track of an earlier position k, up to which an optimal partitioning has been determined. At each step, the algorithm attempts to determine if the window $S_{(k+1)..i}$ should be classified either as an extension of the last known region R (currently extending up to k), or as the prefix of a new region starting at $k + 1$. Depending on the type of region R (included or excluded) and the difference $D = V[I_{(k+1)..i}] - V[X_{(k+1)..i}]$ between the values of the scoring function for the two potential classifications of the window $S_{(k+1)..i}$, the algorithm selects one of three possible options:

1. If R is an included region and D is positive, or if R is an excluded region and D is negative, then R is extended to include the nucleotides up to i (i.e. $k := i$);

2. If R is an included region and D < -C, or if R is an excluded region and D > C, then R is terminated at k and a new region of the opposite type is initialized at $k + 1$ and extended to position i;
3. Otherwise, neither action is taken.

Following this decision, the next nucleotide in the sequence is processed (i.e., i is incremented). The classification of the first and the last regions in the sequence is determined similarly, effectively assuming that the start of the sequence follows an excluded region, and only inspecting the sign of D if R is an included region at the end of the sequence (i.e., when $i = n - 1$).

Since the number of times each nucleotide is examined is bounded by a constant, the overall time complexity is linear with respect to the size of the input sequence. The algorithm runs in constant space, as we need only keep a running value of D, the values of i and k, and the type of region R. A proof of optimality for this algorithm is presented in the Appendix.

This algorithm imposes no implicit upper bound on the size of *nr*DNA partitions, although C is effectively a lower bound on tile sizes. Therefore, included regions must be subdivided into smaller tiles whose sizes reflect the desired upper limit for PCR products. In terms of experimental preparation and subsequent microarray data analysis, it is preferable to create roughly equal-sized fragments whenever possible. Therefore the most straightforward tiling of long *nr*DNA partitions involves 1) taking the ceiling of the length of the partition divided by the maximum tile size, then 2) subdividing the partition into equal-sized fragments of this number.

A further improvement in the time complexity is possible when $u \geq 2l$. In this case it can be shown that for $k > u$, $OptimalTilePath\{S_{1..k}\}$ is always one of the following three tile paths:

1. $OptimalTilePath\{S_{1..(k-l)}\} \cup T_{(k-l+1)..k}$;
2. $OptimalTilePath\{S_{1..(k-1)}\} \cup X_{k..k}$; and
3. $OptimalTilePath\{S_{1..i}\} \cup T_{(i+1)..k}$, where i+1 is the starting index of the last tile in the highest-score path of the form $OptimalTilePath\{S_{1..i}\} \cup T_{(i+1)..k-1}$.

In other words, the best path is the path ending with a tile of minimal length, or ending with a gap, or the one-nucleotide extension of the last tile in the one step shorter best path ending with a tile. Note that the third choice is only valid when $k - i < u$, and indeed it can be shown that when $u \geq 2l$, this constraint is always satisfied; there is always a path ending with a tile shorter than u, whose score is at least that of the path ending with a tile of maximal size u.

As a consequence of this observation, the algorithm can select the best of the above three choices at each nucleotide, instead of comparing $u - l + 2$ alternatives. This reduces the time complexity to $O(n)$. In practice a straightforward implementation of the algorithm finds the optimal tiling of the largest sequenced chromosomes in less than 15 seconds when u=1500 and $u \geq 2l$.

Although the above improvement in time complexity does not hold in general when $u<2l$, in practice the algorithm performs better when applying a similar optimization in the general case: Select the best path among the above three choices, *unless* the third choice is invalid due to the last tile being of length u; in that case, fall back to the previously described algorithm and compute the maximal score among all $u - l + 1$ paths ending with a tile. Experiments with various genome sequences show that the fallback procedure is invoked very rarely, and this optimization makes a significant difference in the running time.

RESULTS

Tiling statistics for eukaryotic genomes

A summary of tiling genome sequences of various sizes and repeat densities is presented in Table 2. Several model organisms were included whose genomes have relatively few repeats, as well as the highly repetitive genomes of more recently sequenced rodents and primates. The sequences were tiled first using a naïve approach where the non-repetitive DNA was subdivided into tiles having lengths equal to the lower size bound (in this case 300 bp). The linear-time tiling method was then applied to the sequences to derive an optimal tile path for each. Table 3 includes a summary of two additional metrics that apply simple tiling schemes to each sequence. Each of the latter methods allows some inclusion of repetitive nucleotides in order to recover a higher percentage of non-repetitive DNA.

In comparing these results, a number of observations become apparent. When the sequences are tiled in a naïve fashion, the coverage of non-repetitive DNA decreases dramatically as we progress from the relatively repeat-free *Arabidopsis* sequence to the larger mammalian genomes. This reflects the higher levels of genomic sequence fragmentation due to increased repeat content, a condition that clearly inhibits the optimal tiling of the sequence. Applying the optimal tiling algorithm to more complex genomes improves the non-repetitive sequence coverage significantly, while the percentage of included repeats remains very low. The optimal tiling algorithm greatly outperforms the other methods in higher eukaryotes, achieving maximal coverage of non-repetitive DNA with a relatively small increase in repeat nucleotide inclusion.

DISCUSSION

Tiling arrays are becoming an important tool for empirical genome annotation, making available the maximum amount of non-repetitive genomic DNA for microarray interrogation. In designing an optimal tile path for microarray applications, the identification and reduction of similar sequences constitutes a fundamental issue and can significantly reduce artifacts associated with cross-hybridization (Royce et al. 2005).

While exact methods can be formulated to address this problem for shorter sequences, such approaches are computationally intractable as sequence tiles become longer. In this case, global approximations based on homology to repetitive elements serve to eliminate redundant sequences on a larger scale.

Numerous options exist for tiling genomic sequences with oligonucleotides, leading to microarray designs of various sequence resolutions and feature densities. Biasing the selection of probes toward uniform thermal properties and eliminating non-unique sequences across the genome can improve the annealing characteristics and hybridization specificity. Although these issues become non-trivial for large genomes, we describe an efficient solution for determining sequence similarity and rejecting non-unique probes that is appropriate for microarray applications.

As sequence tiles increase in size, the sequence fragmentation introduced by repetitive elements reduces the coverage of non-repetitive DNA. For higher eukaryotes, this precludes the use of trivial partitioning strategies where maximal coverage of the non-repetitive sequence is desired. To address this problem we present space- and time-efficient algorithms for generating optimal tile paths to improve the coverage of non-repetitive sequences while minimizing the number of repetitive nucleotides included. In this manner, a greater number of fragments of sufficient size are recovered for amplification, and a higher percentage of non-repetitive DNA is represented on the array. These approaches enable the construction of tiling arrays that maximize the amount of non-repetitive DNA for the discovery of novel functional elements in eukaryotic genomes.

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Appendix: Proof of optimality for the linear-time, constant-space algorithm

To see why this algorithm produces an optimal partitioning, we proceed by induction on the length of the inspected sequence and assume that the algorithm has been correct prior to the *i*th element (i.e. the partitioning up to k is optimal, and no decision can be made so far on the window between $k + 1$ and i). We will show only one case of the proof; the rest is very similar. Without loss of generality assume that the last known region R, currently extending up to k, is an included region. Consider the case when $D < -C$, in which the algorithm will terminate R at i and start an excluded region at $i + 1$. Suppose however that there is an optimal partitioning P with score s_P which extends R at least up to position i, contrary to what the algorithm yields. Define a new partitioning N, identical to P except for the window between $k + 1$ and i, which in N is part of an excluded region, and let us compute its score s_N. There are two possibilities: if in P the included region ends at i and an excluded region starts at $i + 1$, then N has the same number of partitions as P, but one region boundary has been shifted from i in P to k in N. Hence s_N is equal to s_P plus the difference in the scores on the window between $k + 1$ and i; these scores are exactly $V[I_{(k+1)..i}]$ under the partitioning P and $V[X_{(k+1)..i}]$ under N, therefore:

$$s_N = s_P - V[I_{(k+1)..i}] + V[X_{(k+1)..i}] = s_P - D > s_P\,,$$

since the difference D is negative by our assumption. The second possibility is that the included region starting in P extends after i; this means that in N this region is subdivided into two regions by the excluded region from $k + 1$ to i, so N contains one more included region than P. Hence

$$s_N = s_P - V[I_{(k+1)..i}] + V[X_{(k+1)..i}] - 2C \; > s_P$$

again by the assumption that $D < -C$. Thus, in both cases we have $s_N > s_P$, which contradicts the assumption of optimality of the partitioning P.

Other partitionings which terminate the included region earlier than i can be shown similarly suboptimal by the following observation. Since by assumption the algorithm postponed the decision until i, the difference D must be between $-C$ and 0 at all intermediate points. For the case when the algorithm postpones the partitioning decision, the proof of correctness is to construct two sequences sharing the same prefix up to i but requiring different optimal partitionings of the window from $k + 1$ to i, which shows that indeed no decision guaranteeing optimality can be made at i. In other words, a partitioning solution not satisfying the tests in the algorithm cannot be optimal.

The correctness of the optimization is a consequence of the following propositions, which we prove under the assumptions that $u \geq 2l$ and the weights w^T_{rp} and w^X_{nr} are smaller than w^X_{rp} and w^T_{nr} (in particular, without loss of generality we can assume that w^T_{rp} and w^X_{nr} are negative, while w^X_{rp} and w^T_{nr} are positive).

1. For all $k > u$ it holds that $V[OptimalTilePath\{S_{1..k-u+l}\}] \geq V[OptimalTilePath\{S_{1..k-u}\} \cup T_{k-u+1..k-u+l}]$, since the right hand side is the score of one of the paths from which the path in the left hand side is chosen as optimal.
2. Hence, for $k > u$, to find a path of the form $OptimalTilePath\{S_{1..i}\} \cup T_{i+1..k}$ with a maximal score among all $i \in (k-u, k-l]$, it suffices to consider those paths with $i \in (k-2l, k-l]$.
3. Let j be a value of $i \in (k-2l, k-l]$ which maximizes $V[OptimalTilePath\{S_{1..i}\}]$; in this case it also maximizes $V[OptimalTilePath\{S_{1..i}\} \cup T_{i+1..k}]$. To show that this is true, we first assume the contrary:
   a) Consider the possibility that the maximum of $V[OptimalTilePath\{S_{1..i}\} \cup T_{i+1..k}]$ is reached at an index i greater than j, that is, suppose that for some $i \in (j, k-l]$ we have $V[OptimalTilePath\{S_{1..i}\} \cup T_{i+1..k}] > V[OptimalTilePath\{S_{1..j}\} \cup T_{j+1..k}]$. It follows that $V[OptimalTilePath\{S_{1..i}\}] > V[OptimalTilePath\{S_{1..j}\} \cup T_{j+1..i}]$. Then $OptimalTilePath\{S_{1..i}\}$ cannot end with a tile over $S_{j+1..i}$, because in that case its score would, at most, be equal to that of $OptimalTilePath\{S_{1..j}\} \cup T_{j+1..i}$ (which itself covers that subsequence with a tile), instead of strictly greater as assumed. Since $i-j < l$, $OptimalTilePath\{S_{1..i}\}$ cannot end with a tile starting between i and j. Hence it must end with an excluded region; furthermore, the excluded region must start at j+1, otherwise the score of the optimal tile path ending just before the start of the excluded region would be higher than the score of $OptimalTilePath\{S_{1..j}\}$. Consider then the tile path $OptimalTilePath\{S_{1..j}\} \cup X_{j+1..i}$; this gives:

$$V[OptimalTilePath\{S_{1..j}\} \cup X_{j+1..i}] = V[OptimalTilePath\{S_{1..j}\}] + V[X_{j+1..i}],$$

which means this tile path has the same score as $OptimalTilePath\{S_{1..i}\}$ on the subsequence $S_{j+1..i}$, and the highest possible score on $S_{1..j}$, by assumption higher than the corresponding score of $OptimalTilePath\{S_{1..i}\}$. Hence $V[OptimalTilePath\{S_{1..j}\} \cup X_{j+1..i}] > V[OptimalTilePath\{S_{1..i}\}]$, which contradicts the optimality of $OptimalTilePath\{S_{1..i}\}$.

b) Similar reasoning shows that $V[OptimalTilePath\{S_{1..j}\} \cup T_{j+1..k}]$ is higher than the score of any path starting at some $i \in (k\text{-}2l, j)$ and ending with a tile extending to k.

	Contact printing	Inkjet synthesis	Affymetrix	NimbleGen
Arraying method	Mechanical deposition	Phosphoramitide synthesis, piezoelectric printing	In situ DNA synthesis (photolithography)	In situ DNA synthesis (maskless photolithography)
DNA size limit	None	~60 nt	25 nt	~100 nt
Feature type	PCR products, oligomers	Oligomers	Oligomers	Oligomers
Features/slide	<= 40 K	<= 40,000	60 K – 6.2 M	200 – 800 K
Array design	Flexible	Flexible	Fixed	Flexible
Fabrication cost	High (DNA preparation)	Moderate	High	Low
Array cost	Low	Moderate	Moderate	Moderate

Table 1. A comparison of four of the most common DNA microarray formats. While contact-printed arrays allow for unlimited customization, the initial production cost can be prohibitive compared to the relatively affordable but fixed array designs commercially available. Maskless photolithographic arrays represent a trade-off between these platforms, allowing customized design while maintaining very high feature density.

Organism	Genome Size	Percent Repeats	Naïve Partitioning	Optimal Sequence Tiling			Comparison
			Tile Quality	Percent non-repeat bp covered	Percent repeat bp included vs all non-repeat bp	Tile Quality	Percent Improvement
Pan troglodytes	3,083,993,401	57.74	66.05	89.81	4.23	85.58	19.53
Homo sapiens	3,070,537,687	52.38	66.07	89.60	4.06	85.53	19.47
Rattus norvegicus	2,795,745,218	48.75	66.86	91.43	5.54	85.89	19.03
Mus musculus	2,638,213,512	45.62	66.18	91.09	5.51	85.58	19.41
Caenorhabditis elegans	100,277,879	11.26	84.29	98.54	3.10	95.44	11.16
Drosophila melanogaster	129,323,838	14.23	86.89	99.40	2.62	96.78	9.89
Fugu rupripes	349,519,338	15.06	87.97	99.07	2.13	96.94	8.97
Arabidopsis thaliana	119,186,497	0.16	99.97	100.00	0.00	100.00	0.02

Table 2. Optimal and naïve tiling of various sequenced genomes for tile sizes between 300bp and 1.5kb. Repetitive elements were identified using RepeatMasker (Smit and Green, unpublished) and Tandem Repeats Finder (Benson 1999). The genome sequences vary in the degree of repeat density, ranging from mammalian genomes with nearly 50% repeat content to the relatively repeat-free *Arabidopsis* genome. Obtaining a high degree of non-repetitive sequence coverage for the genomes on the latter end of the spectrum is

straightforward. However, as higher eukaryotes are considered it becomes impossible to optimally tile the highly repetitive sequences without further processing.

Organism	Genome Size	Percent Repeats	Case1: Threshold Repeat Inclusion (50bp)			Case 2: Percentage Repeat Inclusion (25%)		
			Percent non-repeat bp covered	Percent repeat bp included vs all non-repeat bp	Tile Quality	Percent non-repeat bp covered	Percent repeat bp included vs all non-repeat bp	Tile Quality
Pan troglodytes	3,083,993,401	57.74	64.85	4.15	62.04	66.85	17.94	52.24
Homo sapiens	3,070,537,687	52.38	65.01	4.09	62.24	67.11	18.22	52.16
Rattus norvegicus	2,795,745,218	48.75	66.66	4.28	63.68	69.42	19.84	52.24
Mus musculus	2,638,213,512	45.62	77.56	4.30	74.07	80.82	20.15	60.43
Caenorhabditis elegans	100,277,879	11.26	89.71	2.18	96.68	99.84	11.12	87.47
Drosophila melanogaster	129,323,838	14.23	97.63	0.03	99.97	100	2.39	97.55
Fugu rupripes	349,519,338	15.06	95.09	1.86	97.74	100	6.33	93.24
Arabidopsis thaliana	119,186,497	0.16	99.51	1.29	98.22	100	13.29	84.68

Table 3. Comparison of two simple tiling metrics that incorporate repetitive nucleotides to improve non-repetitive sequence coverage. In Case 1, repeat sequences less than or equal to 50bp were allowed, and in Case 2 up to 25% of a tile may contain repetitive nucleotides. As in Table 1, tile sizes range from 300bp to 1.5kb. Case 1 achieves only marginal improvement in non-repetitive sequence coverage when compared with the same level of repeat nucleotide inclusion in the optimal tiling case. Non-repetitive sequence coverage in mammalian genomes falls sharply in Case 2 despite the inclusion of a high percentage of repetitive DNA. In each case, performance on mammalian genomes is significantly lower than that of the optimal tiling algorithm (Table 2).

Figure legends

Figure 1. *Left*: Evolution of genomic tiling arrays. Representing large spans of genomic DNA with bacterial artificial chromosome (BAC) clones facilitates global experimentation using relatively few array features, at the expense of low tiling resolution. Higher-resolution designs using PCR products or oligonucleotides allow precise mapping of transcripts and regulatory elements, but require labor-intensive or technologically sophisticated approaches to implement. *Upper right*: Linear feature tiling with gapped and end-to-end oligonucleotide placement. *Lower right*: Overlapping tiles using fractional offset (e.g., one 25mer probe placed every 5nt) and single-base offset placement. The latter strategy provides a finer-resolution tiling of the genomic sequence, and can give a more precise indication of where hybridizing sequences are located on the chromosome.

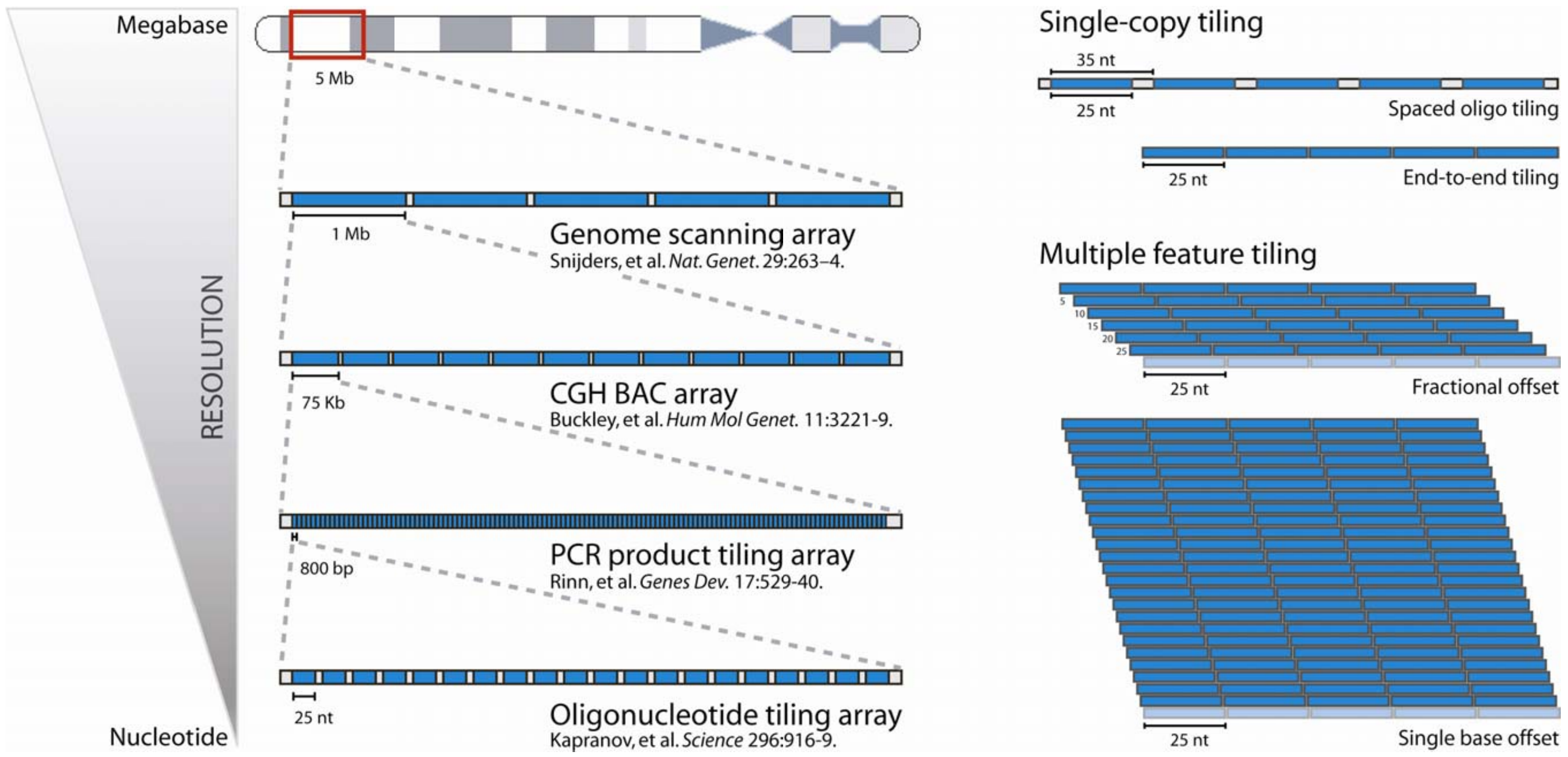
Figure 2. The problem of sequence similarity in tiling genomic DNA. In (**A**), the level of similarity of oligonucleotide sequences to the remainder of the genome is represented by descending bars, where longer bars indicate more redundant sequences. If the redundancy exceeds a given threshold, indicated by the dashed line, the sequence is omitted from the tile path (**B**). Avoiding redundant or repetitive sequences inhibits adequate tiling of the sequence, as shown in **C**. Here, the level of non-repetitive sequence coverage decreases as the minimum tile size increases. At this point it also becomes necessary to use approximations that identify instances of known DNA transposons, retroelements, satellites and other repetitive sequences, rather than calculating an explicit measure of sequence similarity. **D**) In order to recover a higher percentage of non-repetitive DNA, tiling algorithms can be devised that incorporate some redundant sequences, shown in grey, in an optimal fashion which balances the cost of inclusion against the gain in sequence coverage.

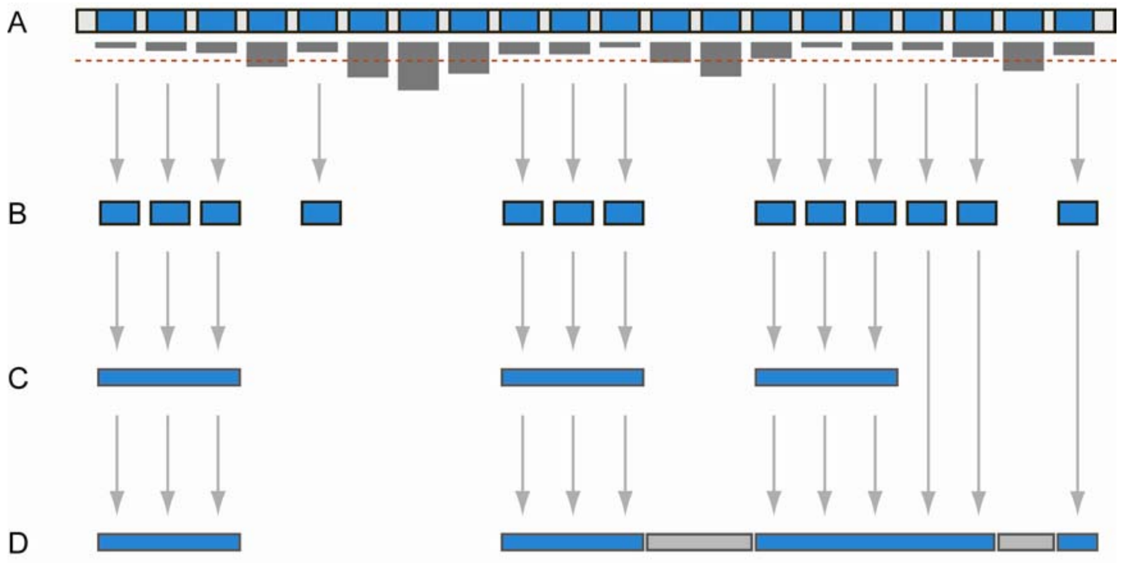
Figure 3. Repeat-masked region of human chromosome 10 showing alternating repetitive and non-repetitive contiguous segments (plotted vertically). The sizes of these subsequences are reflected in the length of the vertical bars. The high level of

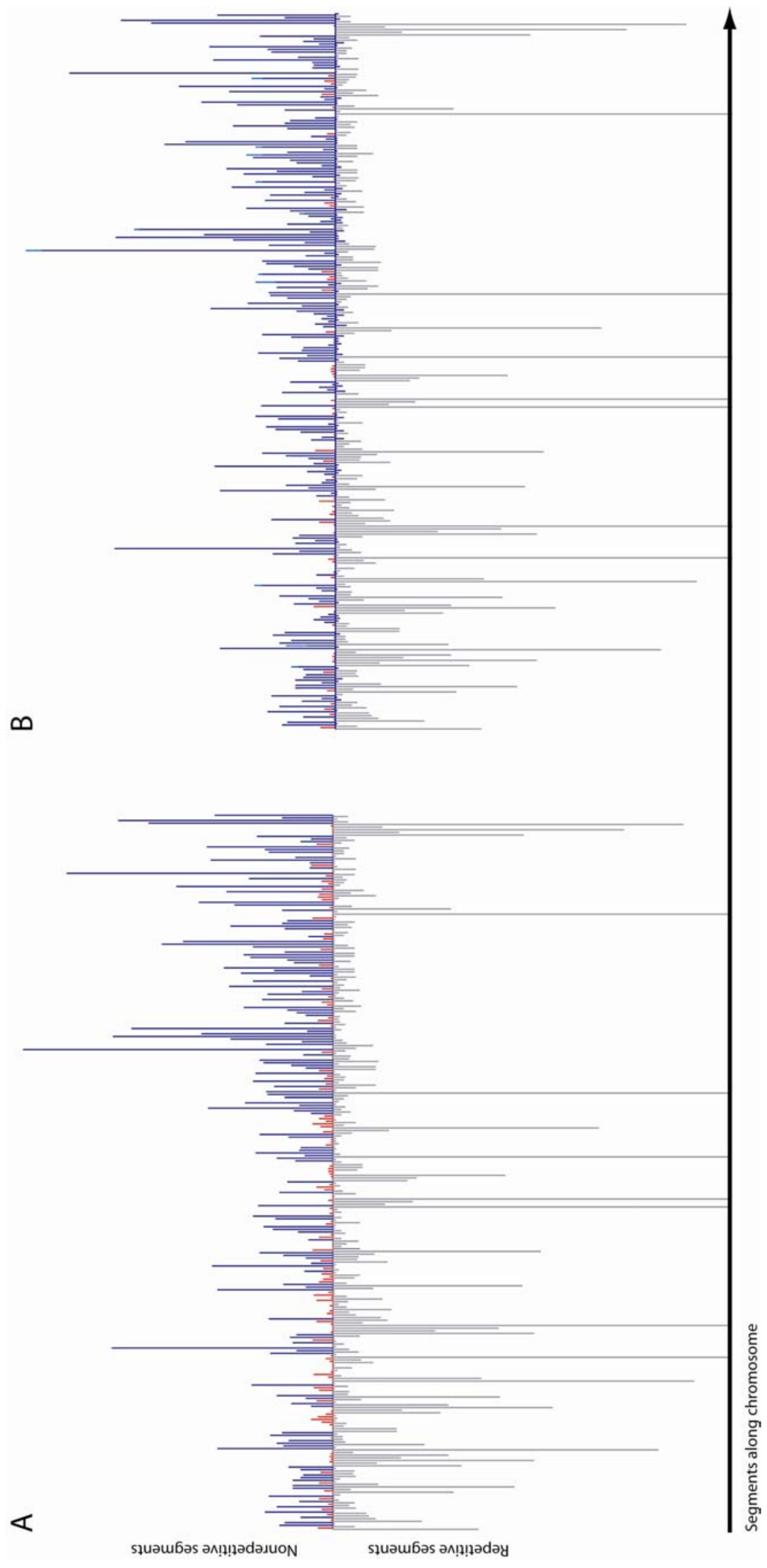
fragmentation is clear, as is the wide range of sizes in both repetitive and non-repetitive sequences. Blue bars depict non-repetitive sequence that is covered in each case, while red bars indicate non-repetitive sequence that is lost. As illustrated in (**A**), a large number of non-repetitive sequences below the minimum size threshold are omitted when using naïve tiling methods that simply avoid repeats. Many of these sequences are recovered after optimal tiling methods are applied (**B**). Note that a small number of repetitive nucleotides are included to increase the coverage, depicted by short blue bars extending below the horizontal line.

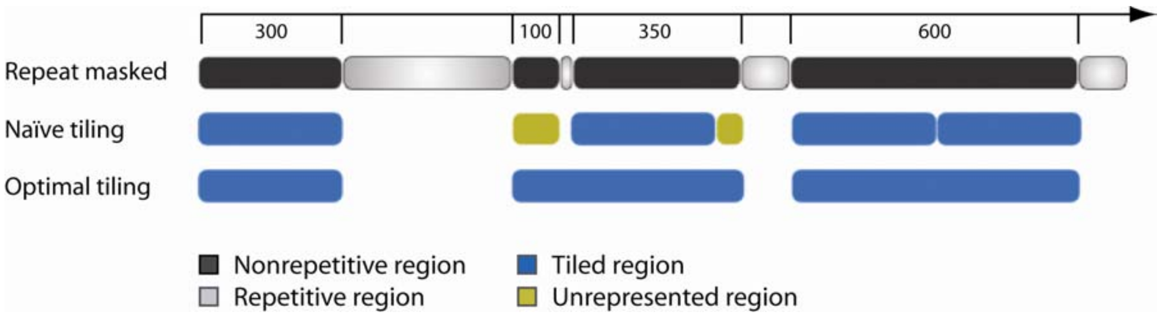
Figure 4. **A**) Graphical representation of repetitive and non-repetitive segments in repeat-masked DNA. In the naïve tiling case (**B**), many small non-repetitive regions might be lost as indicated in yellow. These can be recovered by using partitioning methods that generate an optimal tile path over the sequence (**C**).

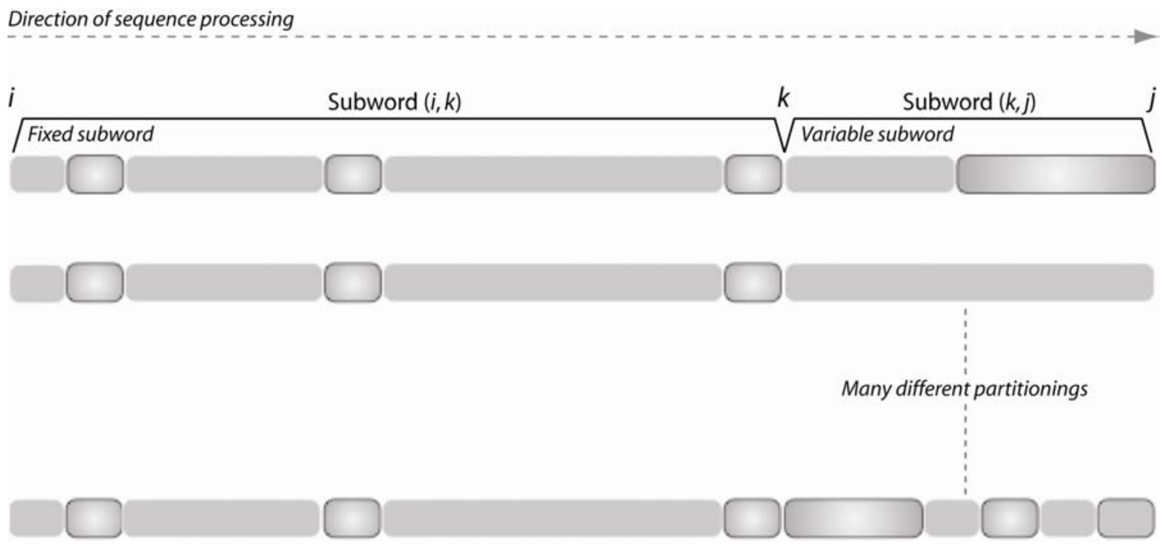
Figure 5. Many different partitionings share common subparts. To compute any partitioning with a split at k, the best partitioning for (i, k) and for (k, j) must be known. Since there are many ways to partition the sequence with a split at k, we only need to recursively evaluate a subpartitioning for subword (i, j) and (k, j) once. In all cases where we need the optimal solution for these subwords again, we refer to the pre-computed result instead of considering all further possible partitionings of that subword.


Megabase
RESOLUTION
Nucleotide
5 Mb
1 Mb
Genome scanning array
Snijders, et al. *Nat. Genet*. 29:263–4.
75 Kb
CGH BAC array
Buckley, et al. *Hum Mol Genet.* 11:3221-9.
800 bp
PCR product tiling array
Rinn, et al. *Genes Dev.* 17:529-40.
25 nt
Oligonucleotide tiling array
Kapranov, et al. *Science* 296:916-9.
Single-copy tiling
35 nt
25 nt
Spaced oligo tiling
25 nt
End-to-end tiling
Multiple feature tiling
5
10
15
20
25
25 nt
Fractional offset
25 nt
Single base offset


A
B
C
D


A
B
Nonrepetitive segments
Repetitive segments
Segments along chromosome


300
100
350
600
Repeat masked
Naïve tiling
Optimal tiling
Nonrepetitive region
Repetitive region
Tiled region
Unrepresented region


Direction of sequence processing
i
Subword (i, k)
k
Subword (k, j)
j
Fixed subword
Variable subword
Many different partitionings