QUIZ 2 CPSC/MBB/MCDB/CBB 452/752a12 Nov. 12, 2012

Name:

Score: /100

1) Given a regulatory network, list at least 4 topological properties that can be calculated. (8 points)

Income degree, outcome degree, path length, network motifs, betweenness centrality, hierarchy, clustering coefficient…

2) What are network hubs? In terms of natural selection, what is the observed characteristic of hubs? (10 points)

Network hubs are nodes with high connectivity. In terms of natural selection, hubs are more conserved.

3) Describe ROC curve. Write down the formulas to calculate sensitivity and specificity. [TP: true positive; FN: false negative; TP: false positive; TN: true negative] (15 points)

ROC curve illustrates the performance of a binary classifier system, as its discrimination threshold is varied. It is created by plotting false positive rates against true positive rates, at various threshold settings. Sensitivity = TP/(TP+FN); Specificity = 1-FP/(FP+TN).

4) List three methods used to find disease modules in biological interaction networks (15 points)

Linkage method; sub-networks; diffusion method.

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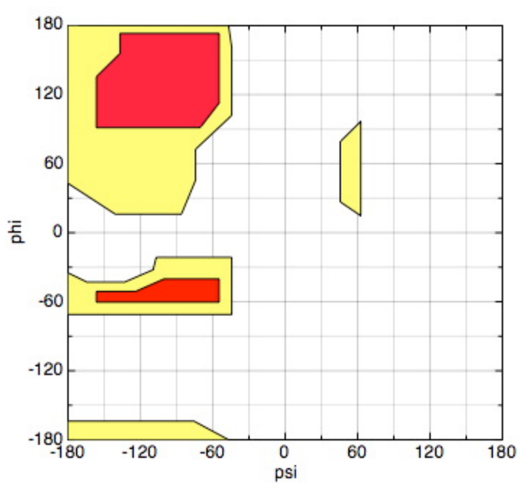
Problem 1:
Calculate the force $\vec{F}_{i,j}$ (magnitude and direction) on atom i due to atom j when they interact via the harmonic spring interaction, $V_s\left(r_{i,j}\right) = \frac{\varepsilon}{2}\left(r_{i,j} - \sigma\right)^2$, where $r_{i,j} = \left|\vec{r}_i - \vec{r}_j\right|$ is the separation between atoms i and j, $\vec{r}_i$ locates the center of atom i, σ is the diameter of the atoms, and ε is the energy scale.

Problem 2: If the normalized probability distribution of atomic speeds v is $P(v) = \left(\frac{1}{2\pi m k_b T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_b T}\right)$, what are $\left\langle v^2 \right\rangle = \left\langle v_x^2 + v_y^2 + v_z^2 \right\rangle$ and $\left\langle v_x^2 \right\rangle$?

Problem 3: Below, a Ramachandran plot based on the original theoretical calculations is shown. Label the regions of the plot that correspond to α-helix and β-sheet backbone conformations and describe roughly what is the difference between the yellow and red regions.



Problem 4: How many degrees of freedom does a polymer possess that contains N spherical monomers that are linked together with N-1 fixed bond lengths.

Quiz 3 Solutions

Problem 5

Remember that $\vec{F} = -\vec{\nabla} V$. These are spherically symmetric potentials, so $\vec{\nabla} V(r) = \frac{\partial V}{\partial r} \hat{r}$. Then we can make $\vec{r}_j$ the origin for the purpose of this problem, so that $\vec{r}_{i,j} = \vec{r}$. Applying this, we get

$$\begin{aligned}\vec{F} &= -\frac{\partial V}{\partial r_{ij}} \vec{r}_{ij} \\ &= \boxed{-\varepsilon \left(r_{i,j} - \sigma\right) \hat{r}_{i,j}}\end{aligned}$$

As it is a spring potential, this is what we expect: it has a magnitude proportional to the spring constant (ε) and the displacement ($r_{i,j} - \sigma$), its in the direction of the vector between the two endpoints ($\hat{r}_{i,j}$), and it has the opposite sign as the displacement.

Problem 6

There are two ways to do this.

Method 1

One is to recognize that this probability distribution is Gaussian $\left(e^{-x^2}\right)$, so we have an energy for v that is quadratic. Then you can remember the equipartition theorem: $\langle E(x) \rangle = \frac{1}{2} k_B T$, where x is the degree of freedom (v_x here), and $E(x)$ is the energy associated with that degree of freedom ($\frac{1}{2} m v_x^2$ here), and then we have

$$\begin{aligned}\left\langle \frac{1}{2} m v_x^2 \right\rangle &= \frac{1}{2} k_B T \\ \left\langle v_x^2 \right\rangle &= \boxed{\frac{k_B T}{m}}\end{aligned}$$

Then $\langle v_x^2 \rangle = \left\langle v_y^2 \right\rangle = \langle v_z^2 \rangle$ because there is nothing "special" about the x, y, and z directions, so

$$\begin{aligned}\left\langle v^2 \right\rangle &= \left\langle v_x^2 + v_y^2 + v_z^2 \right\rangle \\ &= \left\langle v_x^2 \right\rangle + \left\langle v_y^2 \right\rangle + \left\langle v_z^2 \right\rangle \\ &= 3 \left\langle v_x^2 \right\rangle \\ &= \boxed{\frac{3 k_B T}{m}}\end{aligned}$$

Method 2

First, remember that $\langle f(x) \rangle = \int_a^b P(x) f(x)\, dx$, where $f(x)$ is any function, x is a thermodynamic variable, and a and b are the limits. Here we have $x = \vec{v}$, $f(x) = v^2$, and the limits for v are 0 and ∞. So then

$$\begin{aligned}\langle v^2 \rangle &= \iiint v^2 P(\vec{v})\, d^3\vec{v} \\ &= \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} \iiint v^2 e^{-\frac{mv^2}{2k_BT}}\, d^3\vec{v}\end{aligned}$$

Then we can make the substitution $\sqrt{a}\vec{u} = \sqrt{\frac{m}{2k_BT}}\vec{v}$, and then we have

$$\langle v^2 \rangle = \frac{1}{\pi^{3/2}} \frac{2k_BT}{m} a^{5/2} \iiint u^2 e^{-au^2}\, d^3\vec{u}$$

Switch to spherical coordinates, where $u^2 \to r^2$ and $d^3\vec{u} \to 4\pi r^2 dr$:

$$\langle v^2 \rangle = \frac{1}{\pi^{3/2}} \frac{2k_BT}{m} a^{5/2} \int_0^\infty 4\pi r^4 e^{-ar^2}\, dr$$

You can then integrate by parts or use a derivative:

$$\begin{aligned}\langle v^2 \rangle &= \frac{1}{\pi^{3/2}} \frac{2k_BT}{m} a^{5/2} \int_0^\infty 4\pi \frac{\partial^2}{\partial a^2} e^{-ar^2}\, dr \\ &= \frac{1}{\pi^{3/2}} \frac{2k_BT}{m} a^{5/2} \frac{\partial^2}{\partial a^2} \int_0^\infty 4\pi e^{-ar^2}\, dr \\ &= \frac{1}{\pi^{3/2}} \frac{2k_BT}{m} a^{5/2} \frac{\partial^2}{\partial a^2} 4\pi \left(\frac{1}{2}\sqrt{\frac{\pi}{a}}\right) \\ &= \frac{1}{\pi^{3/2}} \frac{2k_BT}{m} a^{5/2} 4\pi \left(\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{\sqrt{\pi}}{a^{5/2}}\right) \\ &= \boxed{\frac{3k_BT}{m}}\end{aligned}$$

And we get the same solution as before.

Problem 7

The upper left region is the β-sheet conformation, and the central left region is for α-helices. The red region is sterically allowed, the yellow region is sterically allowed with smaller radii for the atoms, and the white region is disallowed.

Problem 8

In 3D, every monomer has 3 degrees of freedom, so we start with $3N$ degrees of freedom. Every time you add a constraint, you remove a degree of freedom, so given that we have $N-1$ constraints (the fixed bond lengths), we therefore have $3N - N + 1 = \boxed{2N+1}$ degrees of freedom.